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Maneuvers of motorcycles in queues at signalized intersections

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SUMMARY

The maneuvering models of motorcycles in previous studies often considered motorcycles' traveling in terms of movements in a physical static lane and not in terms of dynamic virtual lane-based movements. For that reason, these models are not able to imitate motorcyclists' behavior well. This paper proposes a maneuverability model framework for motorcycles in queues at signalized intersections with considering the dynamic motorcycle's lane. The model includes (i) a dynamic motorcycle's lane to identify the current, left, and right lanes of the subject motorcycle, (ii) a threshold distance to determine when a motorcyclist starts to consider maneuvering, (iii) a lane selection model to identify the lane preferred by a motorcyclist, and (iv) a gap acceptance model to describe whether or not the lead and lag gaps are acceptable for maneuvering. The model framework captures the variation across the motorcyclist population and over time observations. The models were applied to Hanoi and Hochiminh city, Vietnam, based on microscopic data collected from video images. All of the parameters were estimated using the maximum likelihood method with the statistical estimation software GAUSS. The results show that 77.88% of the observed maneuvers – either staying in the current lane or turning left or right – could be modeled correctly by the proposed models. Copyright © 2010 John Wiley & Sons, Ltd.

KEY WORDS: motorcycle traffic; signalized intersection; traffic operation

1. INTRODUCTION

Heterogeneous traffic with a predominance of motorcycles is very common in many cities in Southeast Asia, where motorization has developed rapidly over the last few decades. In this area, the term "motorcycle dependent city" has been used to indicate a city characterized by low-income neighborhoods, high-density land use, and motorcycle-dominated traffic flow. While many developed countries have typically been facing problems related to four-wheeled traffic, developing countries are facing problems related to small-motorized vehicles. Traffic congestion due to two-wheelers is a serious concern in several cities. Motorcycle traffic, which has very distinguishable characteristics, has a considerable effect on traffic conditions. Motorcycles may reduce the speed of the other modes of transportation and may lead to further congestion because of their shape, small size, and maneuverability. A motorcycle is capable of zigzag maneuvers and can move slowly to the front of the queue in traffic. Further, a motorcycle can impede traffic flow by obstructing the movement of vehicles behind and beside it. The conflicts between motorcycles and other transportation modes, or the conflicts among motorcycles themselves, become more serious at signalized intersections. Motorcycles do not follow the "First In First Out" rule, as other vehicles do. At a typical signalized

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intersection, motorcyclists always attempt to creep slowly to the front of the queue during queue formation or queue discharge. With their flexible maneuverability and much faster response to changing traffic conditions, motorcycles are able to maneuver ahead of vehicle traffic. The reasons for a motorcyclist's maneuvers in a queue may be any or all of the following:

(1) *An attempt to stop at a favorable position during queue formation*: While traveling in a queue during a red-light period, motorcyclists tend to move forward and stop at a position that is closest to the stop line.
(2) *A desire to avoid traveling behind a heavy vehicle*: Due to their preference for a wide and clear field of vision, motorcyclists are more likely to avoid traveling behind a heavy vehicle by maneuvering into another position.
(3) *Preparation for making a turn*: Motorcyclists in an improper position tend to maneuver so that they are in a position that is conducive for making a turn.
(4) *An attempt to avoid an obstruction*: Motorcyclists maneuver to avoid pedestrians crossing the intersection.

The literature shows that some studies have been performed to develop a maneuvering model for motorcycles. Bonte *et al.* [1] constructed a lane-changing algorithm that allows motorcycles to maneuver in heavy traffic. The authors also proposed a virtual lane choice model to express dodging and driving along behavior. In their paper, they hypothesized that motorcyclists do not perceive the road space like other users. However, this assumption is correct only when the proportion of motorcycles in the traffic stream is low. Another study, by Hemakom *et al.* [2], offered an inner city motorcycle behavior model, including a following model and a lane-changing model. In this research, each regular traffic lane was divided into three sub-lanes: left, middle, and right. The lane-changing model explains the meandering behavior of motorcyclists as the act of changing sub-lanes. Then, the following questions arise: What happens when the subject motorcycle moves in the boundary between two sub-lanes and how is the sub-lane identified in such a case? Cho and Wu [3] developed a behavioral model for motorcycle traffic. The model takes into account motorcyclist characteristics, vehicle interactions, and the external environment. However, their model did not include the safe distance and dodging behavior, which occurs commonly in city traffic.

The present study aims to develop a rigorous framework for modeling motorcycle maneuvers in queues at signalized intersections in order to (i) attain a better speed and/or better position and (ii) avoid traveling behind a heavy vehicle. The questions that need to be answered are "when does a rider maneuver?" and "what side does the rider choose?" Other factors that might influence such maneuvering, such as pedestrian movement, turning traffic, and the existence of bus stops, have not been considered in this study.

2. METHODOLOGY

The terminology "motorcycle" used in this research refers to motorized two-wheelers. In Vietnam, the engine capacity of motorcycles, including mopeds, scooters, and normal motorcycles, generally ranges from 50 to 150 cm^3.

In general, the behavior of motorcyclists traveling in a queue can be broadly classified into two regimes: the following regime and the maneuvering regime. When a motorcyclist is sensitive and responds to the actions of the front vehicle, the rider is in the following regime. Alternatively, a rider who tries to maneuver to the left or to the right in order to move into a more favorable position is in the maneuvering regime. This research concentrates exclusively on the maneuvers of motorcycles in queues at signalized intersections, carried out to gain a speed advantage, to move into a better position, or to avoid traveling behind a heavy vehicle. Other tactical maneuvers, such as those related to turning traffic, avoiding pedestrians, *etc.*, have not been taken into consideration.

The methodology for modeling motorcycle maneuvers in this research is based on the lane-changing model for passenger cars, which has been given in the existing literature. The structure of the methodology comprises four main parts, as shown in Figure 1 below.

These four parts can be explained as follows:

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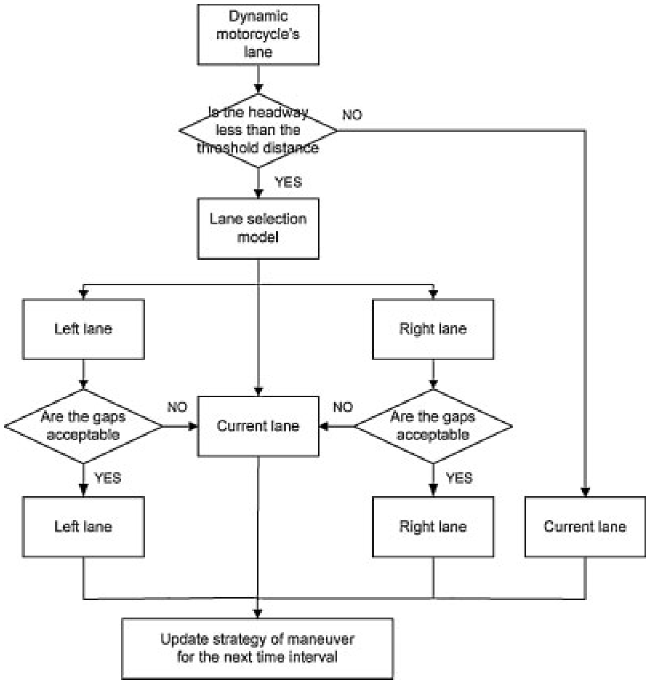

Figure 1. Flowchart of the methodology.

(1) *Dynamic motorcycle's lane*: Because of their specific characteristics, motorcycles are much more flexible than other vehicles and may not follow the same lane discipline as other vehicles do. It is thus essential to introduce a dynamic motorcycle's lane, a virtual lane, to depict the flexible movements of motorcycles.
(2) *Threshold distance for the maneuver model*: The threshold distance for the maneuver model identifies when motorcyclists need to maneuver in order to gain a speed advantage. The distance to the front vehicle is then compared with the threshold distance for the maneuver model. If the distance to the front vehicle is less than the threshold distance for the maneuver model, a motorcyclist starts seeking a better lane. The flowchart then moves to the next step, *i.e.*, the lane selection model. Otherwise, motorcyclists stay in the current lane.
(3) *Lane selection model*: The lane selection model identifies the direction in which a motorcyclist intends to move, such as the current, left, or right lane.
(4) *Gap acceptance model*: The gap acceptance model identifies whether the gaps in the next lane are acceptable for changing lanes or not. If a gap is acceptable, the subject motorcyclist maneuvers to the selected lane; otherwise, the motorcyclist remains in the current lane.

2.1. Dynamic motorcycle's lane

The existing literature shows that motorcycle lanes have been studied in several ways. Sermpis *et al.* [4] and Sermpis and Spyropoulou [5] proposed a lane splitting concept using a special arrangement called ''motorcycle corridors.'' The road was divided into existing and imaginary lanes. Imaginary lanes were located between the existing lanes and/or between the existing lanes and the road infrastructure. However, these studies did not provide any formula or method for estimating the width of the imaginary lane. Another methodology for simulating the moving behavior of motorcycles was adapted by Lan and Chang [6,7] from the cellular automata concept. The width of a lane was divided into cells of equal width (1.25 m). In Hussain *et al.* [8], the physical width of a static motorcycle and the width of the operating space were computed to be 0.8 and 1.3 m, respectively. However, the authors ignored the fact that those values mainly depend on the speed of the motorcycle. In other words, the width of the operating space is larger for a high-speed motorcycle.

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Under Vietnam's traffic conditions, a cross-section of one traffic lane may contain up to three motorcycles. The static lane (one per lane) solution cannot be applied to motorcycle traffic. Therefore, position identification of the subject motorcycle on the roadway should be carried out. The terminology "dynamic motorcycle's lane" with respect to the position of the subject motorcycle is introduced in this study. The dynamic motorcycle's lane is a virtual lane and it is not stable on a roadway like a regular traffic lane; rather, it is flexible according to the subject motorcycle's position. This definition is used only for straight roads, not for curved ones. The dynamic motorcycle's lane takes into account dynamic characteristics of motorcycles. The width of a dynamic motorcycle's lane may be defined as the width of the operating area around the subject motorcycle. This means that when the subject motorcycle moves faster, the width of its lane is wider, and *vice versa*. In this research, the minimum lateral distance between two motorcycles in paired riding is used to obtain the width of the dynamic motorcycle's lane. Paired riding of motorcycles is defined as two motorcycles traveling abreast for over 10 m along a roadway. Hereafter, the word "lane" is used to refer to the dynamic motorcycle's lane.

2.2. Threshold distance for the maneuver model

Motorcyclists do not usually maneuver to the left or right if the vehicle in front is at a sufficient distance, because the rider does not feel any constraint from the front vehicle at that time. It is necessary to determine the threshold distance in order to estimate the time at which the motorcyclists need to maneuver. Hidas [9] described the relationship between the car following model and the lane-changing model by introducing the desired spacing, which was assumed to be a linear function of the speed of the subject vehicle. However, in reality, that distance depends not only on the speed of the subject vehicle but also on the speed of the front vehicle. In this research, the threshold distance for maneuvers is computed using regression analysis. The threshold distance is assumed to be a function of the relative speed with respect to the front vehicle, the occupancy of the motorcycle (number of people on the motorcycle), and the gender of the motorcyclist. Speed data were collected from a video recorder and analyzed using SEV, a video-based software. The gender and other characteristics of the motorcyclists were obtained from video images. At the time of data collection, the helmet law had not been introduced in Vietnam. This means that motorcyclists did not wear helmets while riding motorcycles on the streets at that time. Therefore, it was easy to identify the gender of the subject motorcyclists.

2.3. Lane selection model

The lane selection model describes the willingness of motorcyclists to change lanes. Several studies have been conducted on lane selection models, as given in the existing literature. Wei *et al.* [10] introduced the concepts of speed advantage and speed disadvantage. Regression models of cumulative curves of observed speed advantage and speed disadvantage data were used to simulate the probability of a driver's decision to make a lane change. Webster *et al.* [11] provided a modeling framework for representing the act of planning in the lane changing process. In the model, the driver considered all maneuver sequences over a planning time horizon, and selected the most preferable lane. The model decided the desirability not by considering the current conditions, but by forecasting the resultant states of the subject and surrounding vehicles for various sequences. According to Yang and Koutsopoulos [12], the decision to change lanes was based on the traffic conditions in both the current lane and the adjacent lanes. If the speed of a vehicle were lower than the driver's desired speed, then the driver would check the neighboring lanes for opportunities to increase speed. Several parameters were used to determine whether the current speed was low enough and the speeds in the adjacent lanes were high enough to consider a lane change. However, no mathematical formulation of the proposed model was provided. Ho *et al.* [13] utilized the property of bicycle model to transform the physical lane to a virtual transition lane for the lane changing purpose. The bicycle model calculated the vehicle lateral position during lane change regarding to the virtual transition lane which simulated the lateral position measured by physical sensors for lane keeping. Wu *et al.* [14] applied a fuzzy modeling technique to describe lane-changing behavior. A set of driving rules were compiled and produced for implementation in a fuzzy environment. However, similar to several previous papers, the authors did not show how traffic conditions and driving characteristics affect a lane changing decision. In order

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to overcome this shortcoming, Ahmed [15] and Toledo [16] modeled lane-changing behavior by using a discrete choice framework. Lane change was constructed as a sequence of three steps: the decision to consider a lane change, the choice of the left or right lane, and the search for an acceptable gap to execute the decision. For this study, a similar concept is applied to model the maneuver behavior of motorcycles. The lane selection model describes motorcyclists' willingness to maneuver. The maneuvers of motorcycles to go straight, to the left, or to the right are modeled as the lane changing ability of passenger cars. Since the maneuver decision has three possible answers – the current lane, left lane, or right lane – it can be modeled by using a discrete choice model, *i.e.*, a logit model. The utility functions are as follows:

$$U_n^i(t) = \beta^i X_n^i(t) + \varepsilon_n^i(t); \quad i \in L = \{\mathrm{CL}, \mathrm{RL}, \mathrm{LL}\} \tag{1}$$

where $U_n^i(t)$ is the utility of lane i of motorcyclist n at time t; $\mathbf{X}_n^i(t)$ the vector of explanatory variables of lane i of motorcyclist n at time t; β^i the vector of unknown parameters of lane i; $\varepsilon_n^i(t)$ the random term associated with the lane i of motorcyclist n at time t, which is assumed to be a Gumbel distribution; and CL, RL, LL, are the current lane, right lane, and left lane, respectively.

The choice probability of each lane is as follows:

$$P_n^i(t) = \frac{\exp(V_n^i(t))}{\sum_{j \in L} \exp((V_n^j(t))} \tag{2}$$

where, $P_n^i(t)$ represents the choice probability of lane i of motorcyclist n at time t; and

$V_n^i(t) = \beta^i X_n^i(t)$.

It is clear that a motorcycle moving forward between lead vehicles, even heavy ones, is considered to be choosing the current lane.

2.4. Gap acceptance model

Before maneuvering, a motorcyclist must evaluate both the lead gap and the lag gap; the lead gap is the gap between the motorcycle and the lead vehicle in the preferred lane, and the lag gap is the gap between the motorcycle and the lag vehicle in the preferred lane. If both available gaps are acceptable, then the rider will decide to move into that lane. The gap acceptance model describes whether or not the available gaps in the preferred lane are acceptable. Figure 2 shows an illustration of the lead and lag gaps.

The critical gap is an important factor in the gap acceptance model. It is defined as the minimum acceptable space gap. After selecting the next lane as the preferred lane, the motorcyclist compares the available gaps, *i.e.*, the lead gap and the lag gap, with the corresponding critical gaps. If the available gaps are greater than the corresponding critical gaps, these available gaps are acceptable and the motorcyclist will decide to change lanes. Otherwise, the motorcyclist will stay in the current lane and wait for the next chance.

Although no previous research on a gap acceptance model has been conducted for motorcycle traffic, many similar researches have been carried out for passenger cars and bicycles. Mahmassani and

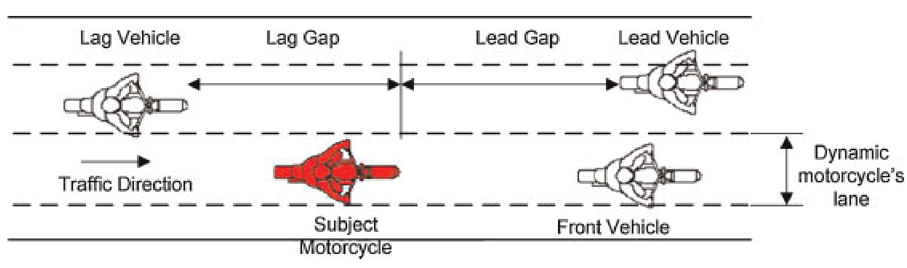

Figure 2. Illustration of lead gap and lag gap for a motorcycle.

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Sheffi [17] estimated the mean and the variance of critical gaps, which were assumed to have a normal distribution, by using a probit-based model. He found that the mean duration of the critical gap was a decreasing function of the number of rejected gaps. Ahmed [15] and Toledo [16] assumed that the driver decides to change lanes only if both gaps, the lead gap and the lag gap, are acceptable. The gap acceptance parameters were estimated jointly using the target lane model. Taylor and Mahmassani [18] estimated the probit models for the gap acceptance decision from observations of behavior of cyclists and motorists when crossing and merging at two-way stop-controlled intersections. They investigated many factors that might affect the mixed-traffic gap acceptance behavior. However, only a small number of observations at just a few low-speed intersections near a university campus were used for the study. In this study, the critical gaps were assumed to be functions of explanatory variables and to follow log normal distributions:

$$\ln(G_n^{\text{cr,lead},i}(t)) = \beta^{\text{cr,lead}}\mathbf{X}_n^{\text{cr,lead},i}(t) + \varepsilon_n^{\text{lead}}(t) \tag{3}$$

$$\ln(G_n^{\text{cr,lag},i}(t)) = \beta^{\text{cr,lag}}\mathbf{X}_n^{\text{cr,lag},i}(t) + \varepsilon_n^{\text{lag}}(t) \tag{4}$$

where $G_n^{\text{cr,lead},i}(t)$ is the critical lead gap acceptance at selected lane i for motorcyclist n at time t (meter); $G_n^{\text{cr,lag},i}(t)$ is the critical lag gap acceptance at selected lane i for motorcyclist n at time t (m); $\mathbf{X}_n^{\text{cr,lead},i}(t)$ and $\mathbf{X}_n^{\text{cr,lag},i}$ are the vectors of the explanatory variables of the lead gap and lag gap at the selected lane i, respectively; $\beta^{\text{cr,lead}}$ and $\beta^{\text{cr,lag}}$ are the vectors of the unknown parameters of the lead gap and lag gap, respectively; and $\varepsilon_n^{\text{lead}}(t)$ and $\varepsilon_n^{\text{lag}}(t)$ are the random terms associated with the lead and lag gaps, respectively. It is assumed that these random terms follow the Gumbel distribution.

$$\varepsilon_n^{\text{lead}}(t) \sim N(0, (\sigma^{\text{lead}})^2), \qquad \varepsilon_n^{\text{lag}}(t) \sim N(0, (\sigma^{\text{lag}})^2)$$

where $(\sigma^{\text{lead}})^2$ and $(\sigma^{\text{lag}})^2$ are the variances of the error terms in the lead and lag gap acceptance models, respectively.

The gap acceptance model assumes that a motorcyclist must accept both the lead available gap ($G_n^{\text{lead},i}(t)$) and the lag available gap ($G_n^{\text{lag},i}(t)$) in the preferred lane i in order to maneuver. Therefore, the probability of the gap acceptance is given by:

$$\begin{aligned} P(\text{GapAcc}|i)_{t,n} &= P(\text{Lead Gap Acceptance}|i)_{t,n} \times P(\text{Lag Gap Acceptance}|i)_{t,n} \\ &= P(G_n^{\text{lead},i}(t) > G_n^{\text{cr,lead},i}(t)) \times P(G_n^{\text{lag},i}(t) > G_n^{\text{cr,lag},i}(t)) \\ &= \Phi\left(\frac{\ln\left(G_n^{\text{lead},i}(t)\right) - G_n^{\text{cr,lead},i}(t)}{\sigma^{\text{lead}}}\right) \times \Phi\left(\frac{\ln\left(G_n^{\text{lag},i}(t)\right) - G_n^{\text{cr,lag},i}(t)}{\sigma^{\text{lag}}}\right) \end{aligned} \tag{5}$$

where $P(\text{GapAcc}/i)_{t,n}$ represents the probability of the gap acceptance at selected lane i for motorcyclist n at time t (m); Φ is the cumulative normal distribution; and $G_n^{\text{lead},i}(t)$ and $G_n^{\text{lag},i}(t)$ are the lead available gap and lag available gap in the selected lane i for motorcyclist n at time t (m).

2.5. *Likelihood function of lane selection model and gap acceptance model*

The lane change sequence for the motorcycle n is described as follows:

$$[(J^i)_{1,n}, (J^i)_{2,n}, \ldots, (J^i)_{In,n}]$$

where J^i is the decision to change to lane i, where $i \in \{\text{CL, RL, LL}\}$; CL, RL, LL are the current lane, right lane, and left lane, respectively; and I_n is the number of observations of motorcyclist n.

The probability of observing a pattern for a given motorcyclist is given by

$$P((J^i)_{1,n}, \ldots, (J^i)_{In,n}) = \prod_{t=1}^{I_n} P((J^{\text{CL}})_{t,n})^{\delta_{t,n}^{\text{CL}}} \times P((J^{\text{RL}})_{t,n})^{\delta_{t,n}^{\text{RL}}} \times P((J^{\text{LL}})_{t,n})^{\delta_{t,n}^{\text{LL}}} \tag{6}$$

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where

$$\delta_{t,n}^{\mathrm{RL}} = \begin{cases} 1 & \text{If motorcyclist } n \text{ changes to right lane at time } t \\ 0 & \text{Otherwise} \end{cases} \quad (7)$$

$$\delta_{t,n}^{\mathrm{LL}} = \begin{cases} 1 & \text{If motorcyclist } n \text{ changes to left lane at time } t \\ 0 & \text{Otherwise} \end{cases} \quad (8)$$

$$\delta_{t,n}^{\mathrm{CL}} = \begin{cases} 1 & \text{If motorcyclist } n \text{ does not change lane at time } t \\ 0 & \text{Otherwise} \end{cases} \quad (9)$$

It is clear that only the final action of a lane change, which lane the motorcycle was in, is observed from the data. The probability of a motorcycle staying in the current lane includes (i) the preferred lane is the current lane and (ii) the preferred lane is the next lane, but the gap, the lead gap, and/or the lag gap in that lane are unacceptable. Therefore, the joint probability of a combination of the preferred lane i and a satisfied condition for a lane change for motorcyclist n at time t is given by

$$P\left((J^i)_{t,n}\right) = \begin{cases} P_n^i(t) \times P(\mathrm{GapAcc}|i)_{t,n} & \text{If } i \text{ is not the current lane} \\ 1-(P_n^{\mathrm{LL}}(t) \times P(\mathrm{GapAcc}|\mathrm{LL})_{t,n} + P_n^{\mathrm{RL}}(t) \times P_n(\mathrm{GapAcc}|\mathrm{RL})_{t,n} & \text{Otherwise} \end{cases} \quad (10)$$

where $P\left((J^i)_{t,n}\right)$ is the joint probability of the chosen lane i and the satisfied condition for a lane change for motorcyclist n at time t.

$P_n^i(t)$ and $P(\mathrm{GapAcc}|i)_{t,n}$ are calculated by (2) and (5) respectively.

Assuming that the lane-change observations of different motorcyclists are independent, the likelihood function for all of the observations of N motorcyclists over time consideration I_n is

$$L = \prod_{n=1}^{N} \prod_{t=1}^{I_n} \left[P((J^{\mathrm{CL}})_{t,n})^{\delta_{t,n}^{\mathrm{CL}}} \times P((J^{\mathrm{RL}})_{t,n})^{\delta_{t,n}^{\mathrm{RL}}} \times P((J^{\mathrm{LL}})_{t,n})^{\delta_{t,n}^{\mathrm{LL}}} \right] \quad (11)$$

where N is the number of observed motorcyclists.

The maximum likelihood estimates for the model parameters were obtained by maximizing the log-likelihood function L. The statistical estimation software GAUSS was used to estimate these parameters. The variables, especially the dummy variables, used to estimate the parameters were coded according to the explanation in Section 4.

3. DATA COLLECTION

With a high population of motorcycles, Hanoi and Hochiminh City, Vietnam, were suitable places to conduct this research. All of the data were collected under the conditions of clear weather, dry pavement, and low magnitude of wind velocity within 10 hours.

The first location, in Hanoi, has three lanes in each direction with a raised median. The widths of the inner, center, and outer lanes are 3.50, 3.64, and 3.37 m, respectively. Fixed-time signalized control has been installed with a cycle time of 130 – 37, 3 , and 90 seconds for the green, yellow, and red times, respectively. There is no all-red time for this signalized intersection. The average queue length is 30 m.

The second location, in Hochiminh City, has two lanes in each direction and no hard median. The inner lane and outer lane are 3.00 and 3.95 m in width, respectively. Most of the vehicle traffic uses the outer lane. The proportion of motorcycles is approximately 80% of the total vehicular traffic. The queue length is about 25 m. This intersection uses fixed-time signalized control with 30, 3, and 40 seconds for the green, yellow, and red times, respectively. It also lacks an all-red time.

The third location, in Hochiminh City, is a one-way street with three lanes. The widths of the inner, middle, and outer lanes are 5.05, 3.85, and 3.15 m, respectively. This location is the most crowded of the three. The average queue length is approximately 43 m. The traffic is a mix of non-motorized

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vehicles, motorcycles, cars, vans, buses, *etc.*; the proportion of motorcycles is more than 85% of the total traffic. The signalized control time is 33, 3, and 37 seconds for the green, yellow, and red times, respectively.

The motorcycle data were collected during both queue formation and discharging at the signalized intersections. Motorcycles in the queues were selected for the study if the distance to the front vehicle was less than the threshold distance for the maneuver model, to ensure that the motorcyclists had already started deciding whether to maneuver. Attributes for the maneuver model, such as the occupancy of the motorcycle; the gender of the rider; spacing to the front vehicles in the current, left, and right lanes; and the relative speed with respect to the leader, were also recorded.

A digital video recorder was set up at the top of a tall building near the study site and captured over 40 m of the roadway. The motorcycles' positions were identified from image video files every one-tenth of a second. These instantaneous positions were calculated according to the screen coordinates, converted into roadway coordinates by using the SEV software, which was developed in the traffic lab for specific purposes. Overall, 150 motorcycles were used to estimate the width of the dynamic motorcycle's lanes. Hundred and four motorcycles were used to estimate the threshold distance. Hundred and eighty-six motorcycles from a total of 558 observations were used for the joint estimation of the parameters of the lane selection model and the gap acceptance model. On average, a motorcycle was observed for 5 second. The data showed that 45.5% of the motorcyclists stayed in their current lane, 37.5% moved into the right lane, and 17% moved into the left lane.

3.1. Data analysis by using SEV software

The computer software SEV, which developed in the traffic lab for this specific purpose, was used in order to analyze the traffic data. The input files were movie clips with a resolution of 640 pixels × 480 pixels, which captured vehicular traffic at the candidate locations. The output files were Excel compatible files, which had advantages when analyzing the trajectory data, as well as other necessary information about motorcycle traffic. SEV has several advantages over conventional counting techniques, as described below:

(1) The ability to measure the trajectories of several vehicles simultaneously.
(2) The ability to measure the position of a vehicle over time intervals as low as one-thirtieth of a second.
(3) The ability to use multiple repetitions to verify the preceding results or recollect missing data, as well as to skim unnecessary data.
(4) Fewer equipment and installation requirements, and few observers required, both on-site and in the laboratory.
(5) It is user-friendly, simple to learn, and easy to operate.

The trajectory data for any vehicle were obtained by clicking on a specific part of that vehicle on the screen at every 0.2 second for more than 10 second of observation time. Then, the motorcycle's trajectory data were used to calculate speeds and the distances between the subject motorcycle and other vehicles by taking the first derivatives of the position with respect to time.

4. ESTIMATION RESULTS

The dynamic motorcycle's lane, the threshold distance for the maneuver model, the lane selection model, and the gap acceptance model were developed. The parameters of each model were estimated and presented in succession as follows.

4.1. Dynamic motorcycle's lane

In this study, the minimum lateral distance between two motorcycles riding abreast was applied to determine the width of the dynamic motorcycle's lane. The minimum lateral distance was assumed to be a function of the average speed. The relationship between a motorcycle's average speed (V) (m/

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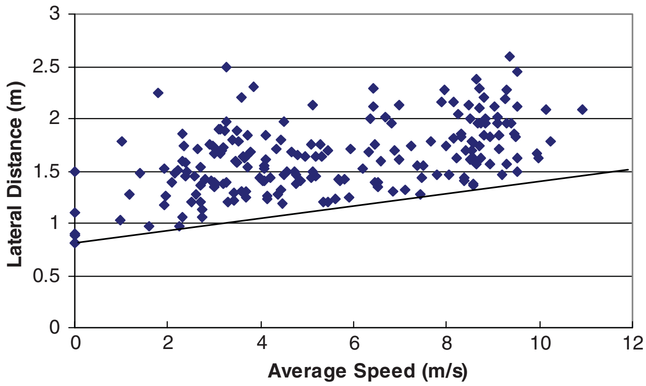

Figure 3. Dynamic lane width of motorcycles.

second) and the motorcycle's dynamic lane width (l_w) (m) may be expressed as

$$l_w = 0.07V + 0.80 \tag{12}$$

where l_w is the motorcycle's dynamic lane width (m) and V is the motorcycle's average speed (m/second).

Figure 3 shows the direct relationship between the average speed and the motorcycle's dynamic lane width. The sample size used to calibrate Equation (12) included 150 motorcycle entities.

4.2. Threshold distance for the maneuver model

The relative speed with respect to the front vehicle (ΔV_n) (m/second) is the significant variable in this model. If the speed of the front vehicle is considerably less than that of the subject motorcycle, the motorcyclist needs a larger distance before starting to maneuver and *vice versa*. This variable is therefore expected to be positive.

The occupancy of the motorcycle also affects the threshold distance. The more the number of people being carried, the lesser is the mobility of the motorcycle. The effect of the occupancy of the motorcycle is indicated by the dummy variable α_n^p. As expected, the estimated coefficient for this variable is greater than 1.

$$\alpha_n^p = \begin{cases} 0 & \text{If only one person is on the motorcycle} \\ 1 & \text{Otherwise} \end{cases} \tag{13}$$

Female motorcyclists usually ride more carefully than male motorcyclists do. The dummy variable α_n^g was introduced to identify the difference between the way male and female motorcyclists ride. This dummy variable is expected to be greater than 1.

$$\alpha_n^g = \begin{cases} 0 & \text{If the motorcyclist is male} \\ 1 & \text{Otherwise} \end{cases} \tag{14}$$

In summary, the threshold distance for the maneuver model, X_n (m), for the maneuver model of the motorcycle n is given by

$$X_n = 1.083 \times \Delta V_n \times 1.188^{\alpha_n^p} \times 1.274^{\alpha_n^g} + 2.001 \tag{15}$$

where ΔV_n is the relative speed to the front vehicle (m/second); α_n^p the occupancy of motorcycle n, defined by (13); α_n^g is the gender of motorcyclist n, defined by (14).

Table I shows the parameter estimates of the threshold distance for the maneuver model. With 104 motorcycle samples, the results indicate that all of the parameters were estimated with a high t-statistic and $R^2 = 0.64$.

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Table I. Parameter estimates of the threshold distance for the maneuver model.

Variable	Estimate	*t*-statistic
Relative speed (m/second) (ΔV_{n})	1.0834	11.170
Number of people dummy (α_n^p)	1.1883	14.180
Gender dummy (α_n^g)	1.2742	13.065
Constant	2.0012	5.263

Number of samples = 104; $R^2 = 0.64$.

4.3. Lane selection model

The variables should reflect (i) individual characteristics, such as the number of people on the motorcycle and the gender of the motorcyclist; and (ii) the conditions for each lane, such as the distance between the front vehicle and the subject motorcycle at each lane and the presence of heavy vehicles.

The constant parameter appears in the utilities of both the current lane and the right lane. Those values are expected to be positive because the left lane, which has a faster speed, may increase the risk of accidents. Moreover, the constant coefficient of the current lane is higher than that of the right lane. This may be explained by the fact that if the other attributes are the same, the current lane is the most preferable.

The effect of the occupancy of the subject motorcycle on the lane utility is introduced as

$$\delta_n^p = \begin{cases} 0 & \text{If only one person is on the motorcycle} \\ 1 & \text{Otherwise} \end{cases} \tag{16}$$

Moreover, in order to express the difference between male and female motorcyclists, the dummy variable δ_n^g is introduced in the model. However, whether or not female motorcyclists prefer the current lane more than male motorcyclists do cannot be determined from the estimation.

$$\delta_n^g = \begin{cases} 0 & \text{If the motorcyclist is male} \\ 1 & \text{Otherwise} \end{cases} \tag{17}$$

The effect of the distance between the subject motorcycle and the front vehicle at lane i, $S_n^{\mathrm{front},i}(t)$ (m), is captured. This parameter identifies how satisfied the motorcyclist is with the conditions in each lane. If this distance increases, the utility with respect to that lane increases as well. Therefore, this parameter is expected to be positive.

The dummy variable captures the motorcyclists' tendency to move out of the current lane if they are following a heavy vehicle. The coefficient of this variable should be negative since it causes inconvenience.

$$\delta_n^{h,i}(t) = \begin{cases} 1 & \text{If the front vehicle of motorcycle } n \text{ at lane } i \text{ at time } t \text{ is a heavy vehicle} \\ 0 & \text{Otherwise} \end{cases} \tag{18}$$

where $i \in L = \{\mathrm{CL}, \mathrm{RL}, \mathrm{LL}\}$.

The lane utility functions are below:

$$U_n^{\mathrm{CL}}(t) = 1.515 + 0.225\delta_n^p + 0.991\delta_n^g + 0.884S_n^{\mathrm{front,CL}}(t) - 3.674\delta_n^{h,\mathrm{CL}}(t) + \varepsilon_n^{\mathrm{CL}}(t) \tag{19}$$

$$U_n^{\mathrm{RL}}(t) = 0.328 + 0.884S_n^{\mathrm{front,CL}}(t) - 3.674\delta_n^{h,\mathrm{RL}}(t) + \varepsilon_n^{\mathrm{RL}}(t) \tag{20}$$

$$U_n^{\mathrm{LL}}(t) = 0.884S_n^{\mathrm{front,LL}}(t) - 3.674\delta_n^{h,\mathrm{LL}}(t) + \varepsilon_n^{\mathrm{LL}}(t) \tag{21}$$

where $U_n^{\mathrm{CL}}(t)$, $U_n^{\mathrm{RL}}(t)$, and $U_n^{\mathrm{LL}}(t)$are the utility function for the current lane, right lane, and left lane of motorcyclist n at time t, respectively; δ_n^p is the occupancy of motorcycle n, defined by (16); δ_n^g the gender of motorcyclist n, defined by (17); $S_n^{\mathrm{front,CL}}(t)$ the distance between motorcycle n and the front vehicle in the current lane; $S_n^{\mathrm{front,RL}}(t)$ the distance between motorcycle n and the lead vehicle in the right lane; $S_n^{\mathrm{front,LL}}(t)$ the distance between motorcycle n and the lead vehicle in the left lane;

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$\delta_n^{h,\mathrm{CL}}(t), \delta_n^{h,\mathrm{RL}}(t), \delta_n^{h,\mathrm{LL}}(t)$ is the existence of a heavy vehicle in the current lane, right lane, and left lane and in front of motorcycle n at time t, respectively, as defined by (18).

4.4. Gap acceptance Model

The function of the explanatory variables includes the constant and the relative speeds of the subject vehicle with respect to the lead or lag vehicles in the selected lane i, *i.e.*, $\Delta V_n^{\mathrm{lead},i}(t)$ (m/second) and $\Delta V_n^{\mathrm{lag},i}(t)$ (m/second), respectively. The magnitudes of both critical gaps depend on the magnitudes of these relative speeds. If the speed of the lead vehicle is considerably higher than that of the subject motorcycle, the motorcyclist is expected to maneuver without requiring a large lead gap. Otherwise, for safety, the motorcyclist needs a large lead gap to complete the lane change. Therefore, the relative speed parameter for the lead critical gap is expected to be negative.

In contrast to the lead critical gap, the lag critical gap increases when the speed of the lag vehicle is considerably higher than that of the subject motorcycle. Inversely, it decreases when the speed of the lag vehicle is a little higher or less than that of the subject motorcycle. Therefore, the relative speed parameter for the lag critical gap should be positive. After estimating the parameters, the gap acceptance models are given by

$$G_n^{\mathrm{cr,lead},i}(t) = \exp(0.602 - 0.295\min(0, \Delta V_n^{\mathrm{lead},i}(t)) + \varepsilon_n^{\mathrm{lead}}(t)) \tag{22}$$

$$G_n^{\mathrm{cr,lag},i}(t) = \exp(-2.343 + 0.732\max(0, \Delta V_n^{\mathrm{lag},i}(t)) + \varepsilon_n^{\mathrm{lag}}(t)) \tag{23}$$

where $G_n^{\mathrm{cr,lead},i}(t)$is the critical lead gap acceptance at selected lane i to motorcyclist n at time t (m); $G_n^{\mathrm{cr,lag},i}(t)$ the critical lag gap acceptance at selected lane i to motorcyclist n at time t (m); $\Delta V_n^{\mathrm{lead},i}(t), \Delta V_n^{\mathrm{lag},i}(t)$ is the relative speeds of motorcycle n with respect to the lead and lag vehicles at selected lane i at time t (m/second), respectively.

Table II shows the parameter estimates of both the lane selection model and the gap acceptance model. With 186 motorcycle samples from a total of 558 observations used for joint estimation of the parameters, the results indicate that most of the parameters were estimated with a high t-statistic.

5. PERFORMANCE OF THE MANEUVER MODEL

A total of 104 maneuvers (not used for calibration) by motorcycles as observed in the trajectory data were compared with the simulated results of the maneuver model. The procedure for testing the maneuver model is explained below:

Table II. Parameter Estimates for Maneuver Models of Motorcycles.

Variable	Estimate	t-statistic
Lane selection model		
CL constant	1.5152	3.192
RL constant	0.3280	1.741
Number of people dummy (δ_n^p)	0.2245	2.513
Gender dummy (δ_n^g)	0.9905	1.340
Front vehicle spacing headway (m) ($S_n^{\mathrm{front},i}$)	0.8835	6.355
Heavy vehicle dummy ($\delta_n^{h,i}$)	−3.6744	−3.922
Lead critical gap		
Constant	0.6017	1.036
Min($\Delta V_n^{\mathrm{lead},i}(t)$, 0) (m/second)	−0.2950	−1.442
σ^{lead}	0.3401	0.640
Lag critical gap		
Constant	−2.3428	−1.571
Max($\Delta V_n^{\mathrm{lag},i}(t)$, 0) (m/second)	0.7323	2.523
σ^{lag}	1.4781	2.514

Number of motorcyclists = 186, Number of observations = 558.
$L(0) = -1217.734$, $L(\beta) = -777.681$, $\bar{\rho}^2 = 0.361$.

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Table III. Differences between observed data and simulated outcomes of the maneuver model.

	Observation			
	Left	Current	Right	Total
Simulation				
Right	7	2	**28**	37
Current	2	**30**	5	37
Left	**23**	1	6	30
Total	32	33	39	**104**

(1) The distance headway to the front vehicle was compared with the threshold distance for the maneuver model (Equation (15)) to ensure that the subject motorcyclist had already started deciding whether to maneuver.
(2) The attributes for the maneuver model, such as the occupancy of the motorcycle, gender of the rider, spacing to the front vehicles in the current, left, and right lanes, and relative speed with respect to the leader, were determined. The disturbances in those models were modeled by generating random variables according to the normal distribution with the mean and standard deviation following Equations (19)–(23).
(3) The observed maneuvers to the left, to the right, or staying in the current lane were compared with the estimation results obtained from the model.

The results are shown in Table III. The numbers in the gray areas indicate the number of maneuvers that could be modeled correctly by the proposed models. The remaining numbers show the mistaken results, *i.e.*, the differences between the simulated outcomes and observed data. The accuracy of the maneuver model with the validated data was 77.88%. This means that 77.88% of the observed maneuvers (either staying in the current lane, or turning left or right) could be modeled correctly by the proposed model.

From the table, the biggest differences between the observed data and simulated results came from the right and left lanes, which were 10.58 and 8.65%, respectively. This means that some motorcyclists preferred to change to either the right or left lane, but the proposed model gave a different lane. This is probably due to the fact that motorcyclists might skip one or more lanes in a single maneuver. For example, even if the right lane's utility is higher than both the utilities of the left and current lanes, a motorcyclist might not decide to maneuver to the right if the utility of the second left lane is higher than that of the right lane and if the available gaps are acceptable for changing to the second left lane. This shortcoming of the present model should be considered in further studies.

6. SUMMARY AND CONCLUSIONS

This paper proposed a maneuverability model framework for motorcycles to simulate when and where motorcycles maneuver in queues at signalized intersections. The concept of lane-changing behavior from passenger cars was adopted. Since motorcycles are flexible and may not follow lane discipline as other vehicles do, an adapted definition for a motorcycle's lane has been introduced. A threshold distance was estimated to identify when motorcycles need to maneuver. The lane selection model, which was developed with a multinomial logit model, was used to determine where motorcycles maneuver. The gap acceptance models were utilized to describe the results of the lane change. The model framework captures the variations in motorcyclist population and over time observations. The models were applied in Hanoi and Hochiminh, Vietnam, based on microscopic data collected from video images. Data related to position, speed, relative speed, and relative spacing with surrounding vehicles were estimated using the SEV software. All of the parameters were estimated using the maximum likelihood method with the statistical estimation software GAUSS. The results showed that 77.88% of the observed maneuvers – either staying in the current lane, or turning left or right – could be modeled correctly using the proposed models. However, the maneuver model has some limitation.

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Since the threshold distance for maneuver model does not include a variable, which captures the surrounding traffic environment, a motorcyclist may have initiated a lane-changing action, but the outcome of the model indicates that this motorcyclist is still in the current lane and takes no action. Moreover, the proposed model in the present paper only explains the maneuverability of motorcycles within three lanes – the current lane, adjacent left lane, and adjacent right lane; it does not simulate a second left lane or a second right lane. Although that kind of movement does not occur regularly, it may cause some errors in the model. These shortcomings of the proposed model should be considered in further studies.

Overall, this study has thrown some new light on motorcycle traffic, with a consideration of dynamic characteristics. The findings of this research will be useful for further researches. According to HCM [20], the criterion used to determine the level of service of exclusive bicycle paths is a weighted sum of the number of passing and paired riding events. The findings of this research can be used to construct the criteria for the level of service for motorcycle paths with a similar concept. The findings can also be applied to estimate the saturation flow rates, as well as the queue lengths where the motorcycle proportion is significant, which are the key components for designing traffic signal systems. Moreover, the knowledge gained from the motorcycle parameters obtained in this research may be used to develop a comprehensive motorcycle simulation model, which will be very valuable for Asian developing countries.

7. LIST OF SYMBOLS AND ABBREVIATIONS

7.1. Roman Case

CL	The current lane
$G_n^{\text{lead},i}(t)$	The lead available gap in the selected lane i for motorcyclist n at time t
$G_n^{\text{lag},i}(t)$	The lag available gap in the selected lane i for motorcyclist n at time t
$G_n^{\text{cr,lead},i}(t)$	The critical lead gap acceptance at selected lane i for motorcyclist n at time t
$G_n^{\text{cr,lag},i}(t)$	The critical lag gap acceptance at selected lane i for motorcyclist n at time t
I_n	The number of observations of motorcyclist n
$J^{\,i}$	The decision to change to lane i
l_w	The motorcycle's dynamic lane width
L	The likelihood function for all of the observations of N motorcyclists over time consideration
LL	The left lane
N	The number of observed motorcyclists
$P_n^i(t)$	The choice probability of lane i of motorcyclist n at time t
$P(\text{GapAcc}\|i)_{t,n}$	The probability of the gap acceptance at selected lane i for motorcyclist n at time t
$P(\text{GapAcc}\|\text{LL})_{t,n}$	The probability of the gap acceptance at the selected left lane for motorcyclist n at time t
$P(\text{GapAcc}\|\text{RL})_{t,n}$	The probability of the gap acceptance at the selected right lane for motorcyclist n at time t
$P(\text{Lead Gap Acceptance}\|i)_{t,n}$	The probability of the lead gap acceptance at selected lane i for motorcyclist n at time t
$P(\text{Lag Gap Acceptance}\|i)_{t,n}$	The probability of the lag gap acceptance at selected lane i for motorcyclist n at time t
$P((J^i)_{t,n})$	The joint probability of the chosen lane i and the satisfied condition for a lane change for motorcyclist n at time t
RL	The right lane
$S_n^{\text{front},i}(t)$	The distance between motorcycle n and the front vehicle in lane i
$S_n^{\text{front,CL}}(t)$	The distance between motorcycle n and the front vehicle in the current lane

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$S_n^{\mathrm{front,RL}}(t)$	The distance between motorcycle n and the lead vehicle in the right lane
$S_n^{\mathrm{front,LL}}(t)$	The distance between motorcycle n and the lead vehicle in the left lane
$U_n^i(t)$	The utility function of lane i of motorcyclist n at time t
$U_n^{\mathrm{CL}}(t)$	The utility function of the current lane of motorcyclist n at time t
$U_n^{\mathrm{RL}}(t)$	The utility function of the right lane of motorcyclist n at time t
$U_n^{\mathrm{LL}}(t)$	The utility function of the left lane of motorcyclist n at time t
V	The motorcycle's average speed
$V_n^i(t)$	The systematic component of the utility function of lane i of motorcyclist n at time t
X_n	The threshold distance for the maneuver model of motorcycle n
$X_n^i(t)$	Vector of explanatory variables of lane i of motorcyclist n at time t
$X_n^{\mathrm{cr,lead},i}(t)$	The vectors of the explanatory variables of the lead gap at the selected lane i of motorcyclist n at time t
$X_n^{\mathrm{cr,lag},i}(t)$	The vectors of the explanatory variables of the lag gap at the selected lane i of motorcyclist n at time t

7.2. Greek Case

α_n^g	Gender dummy parameter of the threshold distance model
α_n^p	Occupancy dummy parameter of the threshold distance model
$\beta^{\,i}$	Vector of unknown parameters of lane i
$\beta^{\,\mathrm{cr,lead}}$	Vector of the unknown parameters of the lead gap
$\beta^{\mathrm{cr,lag}}$	Vector of the unknown parameters of the lag gap
$\delta_{t,n}^{\mathrm{RL}}$	Change lane dummy parameter (specific to the right lane)
$\delta_{t,n}^{\mathrm{LL}}$	Change lane dummy parameter (specific to the left lane)
$\delta_{t,n}^{CL}$	Change lane dummy parameter (specific to the current lane)
δ_n^p	Occupancy dummy parameter of the lane selection model
δ_n^g	Gender dummy parameter of the lane selection model
$\delta_n^{h,i}(t)$	Following-a-heavy-vehicle dummy parameter at lane i at time t
$\delta_n^{h,CL}(t)$	Following-a-heavy-vehicle dummy parameter at the current lane at time t
$\delta_n^{h,RL}(t)$	Following-a-heavy-vehicle dummy parameter at the right lane at time t
$\delta_n^{h,LL}(t)$	Following-a-heavy-vehicle dummy parameter at the left lane at time t
ΔV_n	Relative speed to the front vehicle
$\Delta V_n^{\mathrm{lead},i}(t)$	The relative speed of motorcycle n with respect to the lead vehicle at selected lane i at time t
$\Delta V_n^{\mathrm{lag},i}(t)$	The relative speed of motorcycle n with respect to the lag vehicle at selected lane i at time t
$\varepsilon_n^i(t)$	Random term associated with lane i of motorcyclist n at time t
$\varepsilon_n^{\mathrm{lead}}(t)$	Random term associated with the lead gap of motorcyclist n at time t
$\varepsilon_n^{\mathrm{lag}}(t)$	Random term associated with the lag gap of motorcyclist n at time t
$(\sigma^{\mathrm{lead}})^2$	The variance of the error term in the lead gap acceptance model
$(\sigma^{\mathrm{lag}})^2$	The variance of the error term in the lag gap acceptance model
Φ	The cumulative normal distribution

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7.3. Abbreviations

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